Ejercicios resueltos

1. Calcular los siguientes límites algebraicos

1) $\lim\limits_{x\to 2} \dfrac{x^2+1}{x^2-1} = \dfrac{2^2+1}{2^2-1} = \dfrac{5}{3}$

2) $\lim\limits_{x\to 1} \dfrac{x^2-2x+1}{x^3-x} = \dfrac{1^2-21+1}{1^3-1} = \dfrac{0}{0}$ pero $\lim\limits_{x\to 1} \dfrac{x^2-2x+1}{x^3-x} = \lim\limits_{x\to 1} \dfrac{(x-1)^2}{x(x^2-1)} =$

$\lim\limits_{x\to 1} \dfrac{(x-1)^2}{x(x-1)(x+1)} = \lim\limits_{x\to 1} \dfrac{(x-1)}{x(x+1)} = \dfrac{1-1}{1(2)} = \dfrac{0}{2} = 0$

3) $\lim\limits_{x\to 1} \dfrac{(x-1)\sqrt{2-x}}{1-x^2} = \dfrac{0}{0}$, pero $\lim\limits_{x\to 1} \dfrac{(x-1)\sqrt{2-x}}{1-x^2} = \lim\limits_{x\to 1} \dfrac{(x-1)\sqrt{2-x}}{(1-x)(1+x)} =$

$\lim\limits_{x\to 1} \dfrac{-(1-x)\sqrt{2-x}}{(1-x)(1+x)} = \lim\limits_{x\to 1} \dfrac{-1\sqrt{2-x}}{1+x} = \dfrac{-1}{2}$

4) $\lim\limits_{x\to 1} \dfrac{1}{1-x} - \dfrac{3}{1-x^3}$ $\lim\limits_{x\to 1} \dfrac{1}{1-x} - \dfrac{3}{1-x^3} = \infty - \infty$ que es una forma indeterminada.

Pero $\lim\limits_{x\to 1} \dfrac{1}{1-x} - \dfrac{3}{1-x^3} = \lim\limits_{x\to 1} \dfrac{1}{1-x} - \dfrac{3}{(1-x)(1+x+x^2)} =$

$\lim\limits_{x\to 1} \dfrac{(1+x+x^2)}{1-x} - \dfrac{3)}{(1-x)(1+x+x^2)} = \lim\limits_{x\to 1} \dfrac{(1+x+x^2-3)}{(1-x)(1+x+x^2)} =$

$\lim\limits_{x\to 1} \dfrac{(x^2+x-2)}{(1-x)(1+x+x^2)} = \lim\limits_{x\to 1} \dfrac{(x-1)(x+2)}{(1-x)(1+x+x^2)} =$

$\lim\limits_{x\to 1} \dfrac{-(1-x)(x+2)}{(1-x)(1+x+x^2)} = \lim\limits_{x\to 1} \dfrac{-(x+2)}{(1+x+x^2)} = \dfrac{-3}{3} = -1$

5) $\lim\limits_{x\to 1} \dfrac{x+2}{x^2-5x+4} + \dfrac{x-4}{3(x^2-3x+2)} = \lim\limits_{x\to 1} \dfrac{x+2}{(x-4)(x-1)} + \dfrac{x-4}{3(x-2)(x-1)} =$

$\lim\limits_{x\to 1} \dfrac{3(x+2)(x-2)+(x-4)(x-4)}{3(x-4)(x-2)(x-1)} = \lim\limits_{x\to 1} \dfrac{3(x^2-4)+(x-4)^2)}{3(x-4)(x-2)(x-1)} =$

$\lim\limits_{x\to 1} \dfrac{3x^2-12+x^2-8x+16}{3(x-4)(x-2)(x-1)} = \lim\limits_{x\to 1} \dfrac{4x^2-8x+4}{3(x-4)(x-2)(x-1)} =$

$\lim\limits_{x\to 1} \dfrac{4(x^2-2x+1)}{3(x-4)(x-2)(x-1)} = \lim\limits_{x\to 1} \dfrac{4(x^2-2x+1)}{3(x-4)(x-2)(x-1)} =$

$\lim\limits_{x\to 1} \dfrac{4(x-1)^2}{3(x-4)(x-2)(x-1)} = \lim\limits_{x\to 1} \dfrac{4(x-1)}{3(x-4)(x-2)} = \dfrac{0}{-24} = 0$

6) $\lim\limits_{x\to\infty} \dfrac{2+x-3x^3}{5-x^2+3x^3} = \lim\limits_{x\to\infty} \dfrac{\frac{2+x-3x^3}{x^3}}{\frac{5-x^2+3x^3}{x^3}} = \lim\limits_{x\to\infty} \dfrac{\frac{2}{x^3}+\frac{x}{x^3}-\frac{3x^3}{x^3}}{\frac{5}{x^3}-\frac{x^2}{x^3}+\frac{3x^3}{x^3}} = \dfrac{0+0-3}{0-0+3} = -1$

7) $\lim\limits_{x\to\infty} \dfrac{x^3}{x^2+1} - x = \lim\limits_{x\to\infty} \dfrac{x^3 - x(x^2+1)}{x^2+1} = \lim\limits_{x\to\infty} \dfrac{x^3-x^3-x}{x^2+1} = \lim\limits_{x\to\infty} \dfrac{-x}{x^2+1} =$
$\lim\limits_{x\to\infty} \dfrac{-\frac{x}{x^2}}{\frac{x^2}{x^2}+\frac{1}{x^2}} = \dfrac{0}{1+0} = 0$

8) $\lim\limits_{x\to\infty} \dfrac{\sqrt{x^2+1}-3\sqrt{x}}{\sqrt[4]{x^3+5x}-x} = \lim\limits_{x\to\infty} \dfrac{\frac{\sqrt{x^2+1}-3\sqrt{x}}{x}}{\frac{\sqrt[4]{x^3+5x}-x}{x}} = \lim\limits_{x\to\infty} \dfrac{\frac{\sqrt{x^2+1}}{x}-\frac{3\sqrt{x}}{x}}{\frac{\sqrt[4]{x^3+5x}}{x}-\frac{x}{x}}$
$= \lim\limits_{x\to\infty} \dfrac{\sqrt{\frac{x^2+1}{x^2}}-\frac{3}{x^{1/2}}}{\sqrt[4]{\frac{x^3+5x}{x^4}}-1} = \lim\limits_{x\to\infty} \dfrac{\sqrt{1+\frac{1}{x^2}}-\frac{3}{\sqrt{x}}}{\sqrt[4]{\frac{1}{x}+\frac{5}{x^3}}-1} = \dfrac{\sqrt{1+0}-0}{\sqrt[4]{0+0}-1} = -1$

9) $\lim\limits_{x\to 0} \dfrac{\sqrt{1+x^2}-1}{x}$ Racionalizando obtenemos
$\lim\limits_{x\to 0} \dfrac{\sqrt{1+x^2}-1}{x}\dfrac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}+1} = \lim\limits_{x\to 0} \dfrac{1+x^2-1}{x(\sqrt{1+x^2}+1)} =$
$\lim\limits_{x\to 0} \dfrac{x^2}{x(\sqrt{1+x^2}+1)} = \lim\limits_{x\to 0} \dfrac{x}{\sqrt{1+x^2}+1)} = \dfrac{0}{2} = 0$

10) $\lim\limits_{x\to 5} \dfrac{\sqrt{x-1}-2}{x-5}$ Racionalizando obtenemos
$\lim\limits_{x\to 5} \dfrac{\sqrt{x-1}-2}{x-5}\dfrac{\sqrt{x-1}+2}{\sqrt{x-1}+2} = \lim\limits_{x\to 5} \dfrac{x-1-4}{(x-5)(\sqrt{x-1}+2)} = \lim\limits_{x\to 5} \dfrac{x-5}{(x-5)(\sqrt{x-1}+2)} =$
$\lim\limits_{x\to 5} \dfrac{1}{\sqrt{x-1}+2} = \dfrac{1}{4}$

11) $\lim\limits_{x\to 1} \dfrac{\sqrt[3]{7+x^3}-\sqrt{3+x^2}}{x-1} = \lim\limits_{x\to 1} \dfrac{\sqrt[3]{7+x^3}-\sqrt{3+x^2}+2-2}{x-1} =$
$\lim\limits_{x\to 1} \dfrac{\sqrt[3]{7+x^3}-2}{x-1} - \dfrac{\sqrt{3+x^2}-2}{x-1}$ Racionalizando cada fracción obtenemos:
$\lim\limits_{x\to 1} \dfrac{\sqrt[3]{7+x^3}-2}{x-1}\dfrac{\sqrt[3]{(7+x^3)^2}+2\sqrt[3]{7+x^3}+4}{\sqrt[3]{(7+x^3)^2}+2\sqrt[3]{7+x^3}+4} - \dfrac{\sqrt{3+x^2}-2}{x-1}\dfrac{\sqrt{3+x^2}+2}{\sqrt{3+x^2}+2} =$
$\lim\limits_{x\to 1} \dfrac{(7+x^3)-8}{(x-1)\sqrt[3]{(7+x^3)^2}+2\sqrt[3]{7+x^3}+4)} - \dfrac{3+x^2-4}{(x-1)(\sqrt{3+x^2}+2)} =$
$\lim\limits_{x\to 1} \dfrac{x^3-1}{(x-1)\sqrt[3]{(7+x^3)^2}+2\sqrt[3]{7+x^3}+4)} - \dfrac{x^2-1}{(x-1)(\sqrt{3+x^2}+2)} =$

$$\lim_{x\to 1}\frac{(x-1)(x^2+x+1)}{(x-1)\sqrt[3]{(7+x^3)^2}+2\sqrt[3]{7+x^3}+4)}-\frac{(x-1)(x+1)}{(x-1)(\sqrt{3+x^2}+2)}=$$

$$\lim_{x\to 1}\frac{((x^2+x+1)}{\sqrt[3]{(7+x^3)^2}+2\sqrt[3]{7+x^3}+4)}-\frac{(x+1)}{(\sqrt{3+x^2}+2)}=$$

$$\frac{1+1+1}{4+4+4}-\frac{2}{2+2}=\frac{3}{12}-\frac{2}{4}=\frac{1}{4}-\frac{2}{4}=-\frac{1}{4}$$

12) $\lim_{x\to\infty}\sqrt{(x+m)(x+n)}-x$ Racionalizando queda:

$$\lim_{x\to\infty}\sqrt{(x+m)(x+n)}-x\ \frac{\sqrt{(x+m)(x+n)}+x}{\sqrt{(x+m)(x+n)}+x}=\lim_{x\to\infty}\frac{(x+m)(x+n)-x^2}{\sqrt{(x+m)(x+n)}+x}=$$

$\lim_{x\to\infty}\frac{x^2+(m+n)x+mn-x^2}{\sqrt{(x+m)(x+n)}+x}=\lim_{x\to\infty}\frac{(m+n)x+mn}{\sqrt{x^2+(m+n)x+mn}+x}$ Dividiendo por la

potencia más grande de x, que es x^2 queda:

$$\lim_{x\to\infty}\frac{(m+n)+\frac{mn}{x}}{\sqrt{1+\frac{(m+n)}{x}+\frac{mn}{x^2}}+1}=\frac{(m+n)+0}{\sqrt{1+0+0}+1}=\frac{m+n}{2}$$

13) $\lim_{x\to 2}\frac{1}{2-x}-\frac{3}{8-x^3}=\lim_{x\to 2}\frac{1}{2-x}-\frac{3}{(2-x)(4+2x+x^2)}=\lim_{x\to 2}\frac{4+2x+x^2-3}{(2-x)(4+2x+x^2)}=$

$$\lim_{x\to 2}\frac{x^2+2x+1}{(2-x)(4+2x+x^2)}=\frac{4+4+1}{0}=\infty$$

14) $\lim_{x\to\frac{1}{2}}\frac{8x^2-1}{6x^2-5x+1}=\frac{1}{0}=\infty$

$$\lim_{x\to\frac{1}{2}}\frac{8x^2-2x-1}{6x^2-5x+1}=\lim_{x\to\frac{1}{2}}\frac{(2x-1)(4x+1)}{(3x-1)(2x-1)}=\lim_{x\to\frac{1}{2}}\frac{4x+1}{3x-1}=\frac{2+1}{\frac{3}{2}-1}=\frac{3}{\frac{1}{2}}=6$$

15) $\lim_{x\to\infty}\frac{3x^4-2x+1}{3x^2+6x-2}$ Dividiendo por la potencia más grande de x que es x^4 queda:

$$\lim_{x\to\infty}\frac{3-\frac{2}{x^3}+\frac{1}{x^4}}{\frac{3}{x^2}+\frac{6}{x^3}-\frac{2}{x^4}}=\frac{3}{0}=\infty$$

16) $\lim_{x\to 0}\frac{\sqrt{x^2+4}-2}{\sqrt{x^2+9}-3}=\lim_{x\to 0}\frac{\sqrt{x^2+4}-2}{\sqrt{x^2+9}-3}\ \frac{\sqrt{x^2+4}+2}{\sqrt{x^2+4}+2}\ \frac{\sqrt{x^2+9}+3}{\sqrt{x^2+9}+3}=$

$$\lim_{x\to 0}\frac{(x^2+4-4)(\sqrt{x^2+9}+3)}{(x^2+9-9)(\sqrt{x^2+4}+2)}=\lim_{x\to 0}\frac{(x^2)(\sqrt{x^2+9}+3)}{(x^2)(\sqrt{x^2+4}+2)}=$$

$$\lim_{x\to 0}\frac{\sqrt{x^2+9}+3}{\sqrt{x^2+4}+2}=\frac{\sqrt{0+9}+3}{\sqrt{0+4}+2}=\frac{3+3}{2+2}=\frac{6}{4}=\frac{3}{2}$$

17) $\lim_{x\to 4}\frac{\sqrt{1+2x}-3}{x-4}=\lim_{x\to 4}\frac{\sqrt{1+2x}-3}{x-4}\,\frac{\sqrt{1+2x}+3}{\sqrt{1+2x}+3}=\lim_{x\to 4}\frac{1+2x-9}{(x-4)(\sqrt{1+2x}+3)}=$

$\lim_{x\to 4}\frac{2x-8}{(x-4)(\sqrt{1+2x}+3)}=\lim_{x\to 4}\frac{2(x-4)}{(x-4)(\sqrt{1+2x}+3)}=\lim_{x\to 4}\frac{2}{\sqrt{1+2x}+3}=$
$\frac{2}{6}=\frac{1}{3}$

18) $\lim_{x\to\infty}\frac{3^x+x}{3^x-2x}$ Primero calcularemos $\lim_{x\to\infty}\frac{x}{3^x}$ usando el teorema del Sandwich.
Sabemos que $3^x\geq x^3$ Para $x>3$ Por lo tanto $\frac{x}{3^x}\leq\frac{x}{x^3}$ además $\frac{x}{3^x}\geq\frac{1}{3^x}$ por lo que
podemos afirmar que: $\frac{1}{3^x}\leq\frac{x}{3^x}\leq\frac{x}{x^3}\Rightarrow\frac{1}{3^x}\leq\frac{x}{3^x}\leq\frac{1}{x^2}$ Aplicando límite

$\lim_{x\to\infty}\frac{1}{3^x}\leq\lim_{x\to\infty}\frac{x}{3^x}\leq\lim_{x\to\infty}\frac{1}{x^2}\Rightarrow 0\leq\lim_{x\to\infty}\frac{x}{3^x}\leq 0$ Por lo que $\lim_{x\to\infty}\frac{x}{3^x}=0$

Dividiendo el limite que queremos calcular por 3^x queda:

$$\lim_{x\to\infty}\frac{3^x+x}{3^x-2x}=\lim_{x\to\infty}\frac{1+\frac{x}{3^x}}{1-\frac{2x}{3^x}}=\lim_{x\to\infty}\frac{1+\frac{x}{3^x}}{1-2\frac{x}{3^x}}=\lim_{x\to\infty}\frac{1+0}{1-2(0)}=1$$

19) $\lim_{x\to\infty}\frac{\sqrt[3]{x^2+1}}{x-2}$ Dividiendo por la potencia más grande de x que es x queda:

$$\lim_{x\to\infty}\frac{\sqrt[3]{\frac{x^2}{x^3}+\frac{1}{x^3}}}{\frac{x}{x}-\frac{2}{x}}=\lim_{x\to\infty}\frac{\sqrt[3]{\frac{1}{x}+\frac{1}{x^3}}}{1-\frac{2}{x}}=\frac{0}{1}=0$$

20) $\lim_{x\to\infty}\frac{2x^2+1}{\sqrt[3]{x^7+2x}}$ Dividiendo por la potencia más grande de x que es $x^{7/3}$ queda:

$$\lim_{x\to\infty}\frac{\frac{2x^2}{x^{7/3}}+\frac{1}{x^{7/3}}}{\sqrt[3]{\frac{x^7}{x^7}+\frac{2x}{x^7}}}=\lim_{x\to\infty}\frac{\frac{2}{x^{1/3}}+\frac{1}{x^{7/3}}}{\sqrt[3]{1+\frac{2}{x^6}}}=\frac{0}{1}=0$$

21) $\lim_{x\to 0}\frac{\sqrt{x^2+a^2}-a}{\sqrt{x^2+b^2}-b}=\lim_{x\to 0}\frac{\sqrt{x^2+a^2}-a}{\sqrt{x^2+b^2}-b}\,\frac{\sqrt{x^2+a^2}+a}{\sqrt{x^2+a^2}+a}\,\frac{\sqrt{x^2+b^2}+b}{\sqrt{x^2+b^2}+b}=$

$$\lim_{x\to 0}\frac{(x^2+a^2-a^2)(\sqrt{x^2+b^2}+b)}{(x^2+b^2-b^2)(\sqrt{x^2+a^2}+a)}=\lim_{x\to 0}\frac{x^2(\sqrt{x^2+b^2}+b)}{x^2(\sqrt{x^2+a^2}+a)}=$$

$$\lim_{x\to 0}\frac{\sqrt{x^2+b^2}+b}{\sqrt{x^2+a^2}+a}=\frac{\sqrt{b^2}+b}{\sqrt{a^2}+a}=\frac{2b}{2a}=\frac{b}{a}$$

22) $\lim\limits_{x\to 1} \dfrac{1-\sqrt[3]{x}}{1-\sqrt{x}}$ Sea $u^6 = x$ Si $x \to 1$ entonces $u^6 = 1 \Rightarrow u \to 1$

$\lim\limits_{x\to 1} \dfrac{1-\sqrt[3]{x}}{1-\sqrt{x}} = \lim\limits_{u\to 1} \dfrac{1-\sqrt[3]{u^6}}{1-\sqrt{u^6}} = \lim\limits_{u\to 1} \dfrac{1-u^2}{1-u^3} = \lim\limits_{u\to 1} \dfrac{(1-u)(1+u)}{(1-u)(1+u+u^2)} =$

$\lim\limits_{u\to 1} \dfrac{1+u}{1+u+u^2} = \dfrac{2}{3}$

2. Calcular los siguientes límites trigonométricos

1) $\lim\limits_{x\to 0} \dfrac{sen\,8x}{x} = \lim\limits_{x\to 0} \dfrac{sen\,8x}{x}\dfrac{8}{8} = \lim\limits_{x\to 0} 8\dfrac{sen\,8x}{8x} = (8)(1) = 8$

2) $\lim\limits_{x\to 0} \dfrac{2sen\,(3x)}{5x} = \lim\limits_{x\to 0} \dfrac{2sen\,(3x)}{5x}\dfrac{3}{3} = \lim\limits_{x\to 0} \dfrac{sen\,(3x)}{3x}\dfrac{6}{5} = (1)\left(\dfrac{6}{5}\right) = \dfrac{6}{5}$

3) $\lim\limits_{x\to 0} \dfrac{sen\,(3x)}{sen\,(2x)} = \lim\limits_{x\to 0} \dfrac{sen\,(3x)}{sen\,(2x)}\dfrac{2x}{3x}\dfrac{3}{2} = \lim\limits_{x\to 0} \dfrac{sen\,(3x)}{3x}\dfrac{2x}{sen\,(2x)}\dfrac{3}{2} = (1)(1)\left(\dfrac{3}{2}\right) =$
$\dfrac{3}{2}$

4) $\lim\limits_{x\to 0} \dfrac{tg\,(2x)}{sen\,(5x)} = \lim\limits_{x\to 0} \dfrac{tg\,(2x)}{sen\,(5x)}\dfrac{5x}{2x}\dfrac{2}{5} = \lim\limits_{x\to 0} \dfrac{tg\,(2x)}{2x}\dfrac{5x}{sen\,(5x)}\dfrac{2}{5} = (1)(1)\left(\dfrac{2}{5}\right) = \dfrac{2}{5}$

5) $\lim\limits_{x\to 0} \dfrac{2x}{sen\,(9x)} = \lim\limits_{x\to 0} \dfrac{2x}{sen\,(9x)}\dfrac{7}{7} = \lim\limits_{x\to 0} \dfrac{9x}{sen\,(9x)}\dfrac{2}{9} = (1)\left(\dfrac{2}{9}\right) = \dfrac{2}{9}$

6) $\lim\limits_{x\to 0} \dfrac{sen\,(3x)}{2x} + \dfrac{1}{2} = \lim\limits_{x\to 0} \dfrac{sen\,(3x)}{2x}\dfrac{3}{3} + \dfrac{1}{2} = \lim\limits_{x\to 0} \dfrac{sen\,(3x)}{3x}\dfrac{3}{2} + \dfrac{1}{2} = (1)\left(\dfrac{3}{2}\right) + \dfrac{1}{2} =$
$\dfrac{4}{2} = 2$

7) $\lim\limits_{x\to 0} \dfrac{cos\,(2x) - cos\,(5x)}{x^2} = \lim\limits_{x\to 0} \dfrac{-2sen\left(\frac{2x+5x}{2}\right) sen\left(\frac{2x-5x}{2}\right)}{x^2} =$

$\lim\limits_{x\to 0} \dfrac{-2sen\left(\frac{7x}{2}\right) sen\left(\frac{-3x}{2}\right)}{x^2} = \lim\limits_{x\to 0} \dfrac{sen\left(\frac{7x}{2}\right) sen\left(\frac{-3x}{2}\right)}{\frac{-x^2}{2}} =$

$\lim\limits_{x\to 0} \dfrac{sen\left(\frac{7x}{2}\right)}{x}\dfrac{sen\left(\frac{-3x}{2}\right)}{\frac{-x}{2}} = \lim\limits_{x\to 0} \dfrac{7}{2}\dfrac{sen\left(\frac{7x}{2}\right)}{\frac{7x}{2}}\dfrac{3\,sen\left(\frac{-3x}{2}\right)}{\frac{-3x}{2}} = \dfrac{7}{2}(1)\,(3)(1) = \dfrac{21}{2}$

8) $\lim\limits_{x\to 0} \dfrac{\sqrt{1+sen\,x} - \sqrt{1-tg\,x}}{sen\,(2x)} = \lim\limits_{x\to 0} \dfrac{\sqrt{1+sen\,x} - \sqrt{1-tg\,x}}{sen\,(2x)}\dfrac{\sqrt{1+sen\,x} + \sqrt{1-tg\,x}}{\sqrt{1+sen\,x} + \sqrt{1-tg\,x}} =$

$\lim\limits_{x\to 0} \dfrac{1+sen\,x - (1-tg\,x)}{(sen\,(2x))(\sqrt{1+sen\,x} + \sqrt{1-tg\,x})} = \lim\limits_{x\to 0} \dfrac{sen\,x + tg\,x}{(sen\,(2x))(\sqrt{1+sen\,x} + \sqrt{1-tg\,x})} =$

$$\lim_{x\to 0}\frac{sen\,x+\frac{sex\,x}{cos\,x}}{(sen\,(2x))(\sqrt{1+sen\,x}+\sqrt{1-tg\,x})}=\lim_{x\to 0}\frac{\frac{sen\,x\,cos\,x+sex\,x}{cos\,x}}{(sen\,(2x))(\sqrt{1+sen\,x}+\sqrt{1-tg\,x})}=$$

$$\lim_{x\to 0}\frac{\frac{sen\,x\,(cos\,x+1)}{cos\,x}}{(2sen\,x\,cos\,x)(\sqrt{1+sen\,x}+\sqrt{1-tg\,x})}=$$

$$\lim_{x\to 0}\frac{sen\,x\,(cos\,x+1)}{(2sen\,x\,cos^2\,x)(\sqrt{1+sen\,x}+\sqrt{1-tg\,x})}=\lim_{x\to 0}\frac{cos\,x+1}{(2\,cos^2\,x)(\sqrt{1+sen\,x}+\sqrt{1-tg\,x})}=$$

$$\frac{cos\,0+1}{(2\,cos^2\,0)(\sqrt{1+sen\,0}+\sqrt{1-tg\,0})}=\frac{1+1}{2(1)(\sqrt{1+0}+\sqrt{1-0})}=\frac{2}{4}=\frac{1}{2}$$

9) $$\lim_{x\to 0}\frac{1}{sen\,x}-\frac{1}{tg\,x}=\lim_{x\to 0}\frac{1}{sen\,x}-\frac{1}{\frac{sen\,x}{cos\,x}}=\lim_{x\to 0}\frac{1}{sen\,x}-\frac{cos\,x}{sen\,x}=\lim_{x\to 0}\frac{1-cos\,x}{sen\,x}=$$

$$\lim_{x\to 0}\frac{1-cos\,x}{sen\,x}\,\frac{1+cos\,x}{1+cos\,x}=\lim_{x\to 0}\frac{1-cos^2\,x}{sen\,x(1+cos\,x)}=\lim_{x\to 0}\frac{sen^2\,x}{sen\,x(1+cos\,x)}=$$

$$\lim_{x\to 0}\frac{sen\,x}{1+cos\,x}=\frac{0}{2}=0$$

10) $$\lim_{x\to\frac{\pi}{2}}\left(\frac{\pi}{2}-x\right)tg\,x$$

Sea $u=\frac{\pi}{2}-x\Rightarrow x=\frac{\pi}{2}-u$. Si $x\to\frac{\pi}{2}$ entonces $u\to 0$

$$\lim_{x\to\frac{\pi}{2}}\left(\frac{\pi}{2}-x\right)tg\,x=\lim_{u\to 0}u\,tg\left(\frac{\pi}{2}-u\right)=\lim_{u\to 0}u\frac{sen\,(\frac{\pi}{2}-u)}{cos\,(\frac{\pi}{2}-u)}=$$

$$\lim_{u\to 0}u\,\frac{sen\frac{\pi}{2}\,cos\,u-sen\,u\,cos\frac{\pi}{2}}{cos\frac{\pi}{2}\,cos\,u+sen\frac{\pi}{2}\,sen\,u}=\lim_{u\to 0}u\,\frac{cos\,u}{sen\,u}=\lim_{u\to 0}cos\,u\,\frac{u}{sen\,u}=(1)(1)=1$$

11) $\displaystyle\lim_{x\to 0}\frac{3x-arcsen\,x}{3x+arctg\,x}$ Por infinitesimales sabemos que $arcsen\,x\approx x$ y $arctg\,x\approx x$ por lo que:

$$\lim_{x\to 0}\frac{3x-arcsen\,x}{3x+arctg\,x}=\lim_{x\to 0}\frac{3x-x}{3x+x}=\lim_{x\to 0}\frac{2x}{4x}=\frac{2}{4}=\frac{1}{2}$$

12) $$\lim_{x\to 0}\frac{\sqrt{2}-\sqrt{1+cos\,x}}{sen^2\,x}=\lim_{x\to 0}\frac{\sqrt{2}-\sqrt{1+cos\,x}}{sen^2\,x}\,\frac{\sqrt{2}+\sqrt{1+cos\,x}}{\sqrt{2}+\sqrt{1+cos\,x}}=$$

$$\lim_{x\to 0}\frac{2-(1+cos\,x)}{(sen^2\,x)(\sqrt{2}+\sqrt{1+cos\,x})}=\lim_{x\to 0}\frac{1-cos\,x}{(sen^2\,x)(\sqrt{2}+\sqrt{1+cos\,x})}=$$

$$\lim_{x\to 0}\frac{1-cos\,x}{(sen^2\,x)(\sqrt{2}+\sqrt{1+cos\,x})}\,\frac{1+cos\,x}{1+cos\,x}=\lim_{x\to 0}\frac{1-cos^2\,x}{(1+cos\,x)(sen^2\,x)(\sqrt{2}+\sqrt{1+cos\,x})}=$$

$$\lim_{x\to 0}\frac{sen^2 x}{(1+cos\,x)(sen^2 x)(\sqrt{2}+\sqrt{1+cos\,x})}=\lim_{x\to 0}\frac{1}{(1+cos\,x)(\sqrt{2}+\sqrt{1+cos\,x})}=$$

$$\frac{1}{(2)(\sqrt{2}+\sqrt{2})}=\frac{1}{4\sqrt{2}}=\frac{\sqrt{2}}{8}$$

13) $$\lim_{x\to 0}\frac{cos\,(a+x)-cos\,(a-x)}{x}=\lim_{x\to 0}\frac{-2sen\,\left(\frac{(a+x)+(a-x)}{2}\right)\,sen\,\left(\frac{(a+x)-(a-x)}{2}\right)}{x}=$$
$$\lim_{x\to 0}\frac{-2sen\,\left(\frac{2a}{2}\right)\,sen\,\left(\frac{2x}{2}\right)}{x}=\lim_{x\to 0}\frac{-2sen\,(a)\,sen\,(x)}{x}=\lim_{x\to 0}-2sen\,(a)\frac{sen\,(x)}{x}=$$
$$-2sen\,(a)$$

14) $\lim_{x\to 0} cosec\,x-cotg\,x=\lim_{x\to 0}\frac{1}{sen\,x}-\frac{1}{tg\,x}=0$ Resuelto con anterioridad.

15) $$\lim_{x\to\frac{\pi}{4}}\frac{cotg\,x-tg\,x}{sen\,x-cos\,x}=\lim_{x\to\frac{\pi}{4}}\frac{\frac{cos\,x}{sen\,x}-\frac{sen\,x}{cos\,x}}{sen\,x-cos\,x}=\lim_{x\to\frac{\pi}{4}}\frac{\frac{cos^2 x-sen^2 x}{sen\,x\,cos\,x}}{sen\,x-cos\,x}=$$
$$\lim_{x\to\frac{\pi}{4}}\frac{cos^2 x-sen^2 x}{(sen\,x-cos\,x)(sen\,x\,cos\,x)}=\lim_{x\to\frac{\pi}{4}}\frac{(cos\,x-sen\,x)(cos\,x-sen\,x)}{(sen\,x-cos\,x)(sen\,x\,cos\,x)}=$$
$$\lim_{x\to\frac{\pi}{4}}\frac{-(cos\,x-sen\,x)}{sen\,x\,cos\,x}=\frac{-(\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2})}{\frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2}}=0$$

16) $\lim_{x\to 0}\frac{cos\,(2x)+cotg\,x+sen\,x}{cos\,x}$ Indeterminado. ¿Mal copiado?

17) $$\lim_{x\to 0}\frac{cos\,(2x)-sen\,(\frac{\pi}{2}-x)}{x}=\lim_{x\to 0}\frac{cos\,(2x)-cos\,x}{x}=\lim_{x\to 0}\frac{cos^2 x-sen^2 x-cos\,x}{x}=$$

$$\lim_{x\to 0}\frac{cos\,x(cos\,x-1)-sen^2 x}{x}=\lim_{x\to 0}\frac{cos\,x(cos\,x-1)-sen^2 x}{x}\;\frac{cos\,x+1}{cos\,x+1}=$$
$$\lim_{x\to 0}\frac{cos\,x(cos^2 x-1)-(cos\,x+1)sen^2 x}{x(cos\,x+1)}=\lim_{x\to 0}\frac{cos\,x(sen^2 x)-(cos\,x+1)sen^2 x}{x(cos\,x+1)}=$$
$$\lim_{x\to 0}\frac{(sen^2 x)(cos\,x-(cos\,x+1)}{x(cos\,x+1)}=\lim_{x\to 0}\frac{-(sen^2 x)}{x(cos\,x+1)}=\lim_{x\to 0}\frac{-sen\,x}{x}\frac{sen\,x}{cos\,x+1}=$$
$$(-1)\left(\frac{0}{2}\right)=0$$

18) $$\lim_{x\to\frac{\pi}{2}}\frac{\sqrt{cos\,x}-cotg\,x}{sen\,(2x)}=\lim_{x\to\frac{\pi}{2}}\frac{\sqrt{cos\,x}-\frac{cos\,x}{sen\,x}}{sen\,(2x)}=\lim_{x\to\frac{\pi}{2}}\frac{\sqrt{cos\,x}-\frac{cos\,x}{sen\,x}}{2\,sen\,x\,cos\,x}=$$

$$\lim_{x\to\frac{\pi}{2}} \frac{\frac{sen\,x\sqrt{cos\,x}-cos\,x}{sen\,x}}{2\,sen\,x\,cos\,x} = \lim_{x\to\frac{\pi}{2}} \frac{sen\,x\sqrt{cos\,x}-cos\,x}{2\,sen^2\,x\,cos\,x} = \lim_{x\to\frac{\pi}{2}} \frac{sen\,x\sqrt{cos\,x}}{2\,sen^2\,x\,cos\,x} - \frac{cos\,x}{2\,sen^2\,x\,cos\,x} =$$

$$\lim_{x\to\frac{\pi}{2}} \frac{1}{2\,sen\,x\,\sqrt{cos\,x}} - \frac{1}{2\,sen^2\,x} = \frac{1}{0} - \frac{1}{2} = \infty$$

19) $$\lim_{x\to 0} \frac{\sqrt{1+tg\,x}-\sqrt{1-tg\,x}}{tg\,(2x)} = \lim_{x\to 0} \frac{\sqrt{1+tg\,x}-\sqrt{1-tg\,x}}{tg\,(2x)}\,\frac{\sqrt{1+tg\,x}+\sqrt{1-tg\,x}}{\sqrt{1+tg\,x}+\sqrt{1-tg\,x}} =$$

$$\lim_{x\to 0} \frac{1+tg\,x-(1-tg\,x)}{(\sqrt{1+tg\,x}+\sqrt{1-tg\,x})tg\,(2x)} = \lim_{x\to 0} \frac{2tg\,x}{(\sqrt{1+tg\,x}+\sqrt{1-tg\,x})tg\,(2x)} =$$

$$\lim_{x\to 0} \frac{2}{\sqrt{1+tg\,x}+\sqrt{1-tg\,x}}\,\frac{tg\,x}{tg\,(2x)}\,\frac{x}{x} =$$

$$\lim_{x\to 0} \frac{1}{\sqrt{1+tg\,x}+\sqrt{1-tg\,x}}\,\frac{tg\,x}{x}\,\frac{2x}{tg\,(2x)} = \frac{1}{\sqrt{1+0}+\sqrt{1-0}}(1)(1) = \frac{1}{2}$$

20) $$\lim_{x\to 0} \frac{sen^2\,x-3tg^4\,x}{sen^3\,x-sen^2\,x} = \lim_{x\to 0} \frac{(sen^2\,x)(1-3(tg^2\,x\,sec^2\,x))}{sen^2\,x(sen\,x-1)} =$$

$$\lim_{x\to 0} \frac{1-3(tg^2\,x\,sec^2\,x)}{sen\,x-1} = \frac{1-3(0*1)}{0-1} = -1$$

3. Calcular los siguientes límites exponenciales y logarítmicos

1) $\lim_{x\to 0}(1+2x)^{\frac{1}{x}} = \lim_{x\to 0}[(1+2x)^{\frac{1}{2x}}]^2 = e^2$

2) $\lim_{x\to 0} \frac{e^{3x}-1}{x} = 3\frac{e^{3x}-1}{3x} = 3(1) = 3$

3) $\lim_{x\to 0}(1-2x)^{\frac{1}{x}} = \lim_{x\to 0}(1+(-2x))^{\frac{1}{x}} = \lim_{x\to 0}[(1+(-2x))^{\frac{1}{-2x}}]^{-2} = e^{-2}$

4) $\lim_{x\to 0}(1+3x)^{\frac{2}{x}} = \lim_{x\to 0}[(1+3x)^{\frac{1}{x}}]^2 = \lim_{x\to 0}[(1+3x)^{\frac{1}{3x}}]^6 = e^6$

5) $$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{4x} = \lim_{x\to\infty}\left[\left(1+\frac{1}{x}\right)^{x}\right]^4 = e^4$$

6) $$\lim_{x\to\infty}\left(1+\frac{1}{2x}\right)^{5x+3} = \lim_{x\to\infty}\left[\left(1+\frac{1}{2x}\right)^{2x}\right]^{\dfrac{5x+3}{2x}} =$$

$$\left[\lim_{x\to\infty}\left(1+\frac{1}{2x}\right)^{2x}\right]^{\lim_{x\to\infty}\dfrac{5x+3}{2x}} = e^{\frac{5}{2}}$$

7) $\lim\limits_{x\to 0}\dfrac{e^{tg\,x}-1}{tg\,x}$ Sea $u=tg\,x$ si $x\to 0$ entonces $u\to 0$ y $\lim\limits_{x\to 0}\dfrac{e^{tg\,x}-1}{tg\,x}=$ $\lim\limits_{u\to 0}\dfrac{e^{u}-1}{u}=1$

8) $\lim\limits_{x\to 0}\dfrac{e^{2x}-1}{sen\,(3x)}=\lim\limits_{x\to 0}\dfrac{e^{2x}-1}{sen\,(3x)}\;\dfrac{3x}{2x}\;\dfrac{2}{3}=\lim\limits_{x\to 0}\dfrac{e^{2x}-1}{2x}\;\dfrac{3x}{sen\,(3x)}\;\dfrac{2}{3}=(1)(1)\left(\dfrac{2}{3}\right)=$ $\dfrac{2}{3}$

9) $\lim\limits_{x\to 0}\dfrac{e^{3x}-e^{x}}{arctg\,x}$ Por infinitesimales sabemos que $arctg\,x\approx x$ entonces

$\lim\limits_{x\to 0}\dfrac{e^{3x}-e^{x}}{arctg\,x}=\lim\limits_{x\to 0}\dfrac{e^{3x}-e^{x}}{x}=\lim\limits_{x\to 0}\dfrac{e^{3x}-1-(e^{x}-1)}{x}=$

$\lim\limits_{x\to 0}\dfrac{e^{3x}-1}{x}-\dfrac{e^{x}-1}{x}=3\dfrac{e^{3x}-1}{3x}-\dfrac{e^{x}-1}{x}=3(1)-1=2$

10) $\lim\limits_{x\to 0}\dfrac{e^{5x}-e^{2x}}{arctg\,(3x)-arctg\,(2x)}$ Por infinitesimales $arctg\,(3x)\approx 3x$ y $arctg\,(2x)\approx$ $2x$

$\lim\limits_{x\to 0}\dfrac{e^{5x}-e^{2x}}{3x-2x}=\lim\limits_{x\to 0}\dfrac{e^{5x}-1-(e^{2x}+1)}{x}=\lim\limits_{x\to 0}\dfrac{e^{5x}-1}{x}-\dfrac{e^{2x}-1}{x}=$

$\lim\limits_{x\to 0}5\dfrac{e^{5x}-1}{5x}-2\dfrac{e^{2x}-1}{2x}=5(1)-2(1)=3$

11) $\lim\limits_{x\to 0}\dfrac{arcsen\,x-arctg\,x}{x^{3}}$

Sea $\alpha=arcsen\,x$, $\beta=arctg\,x$ y $t=arcsen\,x-arctg\,x\Rightarrow t=\alpha-\beta\Rightarrow$

$sen\,t=sen\,(\alpha-\beta)=sen\,\alpha\,cos\,\beta-cos\,\alpha\,sen\,\beta$

Como $\alpha=arcsen\,x\Rightarrow sen\,\alpha=x, cos\,\alpha=\sqrt{1-x^{2}}$ y

$\beta=arctg\,x\Rightarrow tg\,\beta=x\Rightarrow sen\,\beta=\frac{x}{\sqrt{x^{2}+1}}\wedge cos\,\beta=\frac{1}{\sqrt{x^{2}+1}}\Rightarrow$

$sen\,t=x\frac{1}{\sqrt{x^{2}+1}}-\sqrt{1-x^{2}}\frac{x}{\sqrt{x^{2}+1}}\Rightarrow t=arcsen\left(x\frac{1}{\sqrt{x^{2}+1}}-\sqrt{1-x^{2}}\frac{x}{\sqrt{x^{2}+1}}\right)$

Reemplazando:

$$\lim_{x\to 0}\frac{arcsen\left(x\frac{1}{\sqrt{x^{2}+1}}-\sqrt{1-x^{2}}\frac{x}{\sqrt{x^{2}+1}}\right)}{x^{3}}\;\frac{x\frac{1}{\sqrt{x^{2}+1}}-\sqrt{1-x^{2}}\frac{x}{\sqrt{x^{2}+1}}}{x\frac{1}{\sqrt{x^{2}+1}}-\sqrt{1-x^{2}}\frac{x}{\sqrt{x^{2}+1}}}=$$

$$\lim_{x\to 0}\frac{arcsen\left(x\frac{1}{\sqrt{x^{2}+1}}-\sqrt{1-x^{2}}\frac{x}{\sqrt{x^{2}+1}}\right)}{x\frac{1}{\sqrt{x^{2}+1}}-\sqrt{1-x^{2}}\frac{x}{\sqrt{x^{2}+1}}}\;\frac{x\frac{1}{\sqrt{x^{2}+1}}-\sqrt{1-x^{2}}\frac{x}{\sqrt{x^{2}+1}}}{x^{3}}=$$

$$\lim_{x\to 0}\frac{arcsen\left(x\frac{1}{\sqrt{x^2+1}}-\sqrt{1-x^2}\frac{x}{\sqrt{x^2+1}}\right)}{x\frac{1}{\sqrt{x^2+1}}-\sqrt{1-x^2}\frac{x}{\sqrt{x^2+1}}}\;\frac{\frac{x}{\sqrt{x^2+1}}(1-\sqrt{1-x^2})}{x^3}=$$

$$\lim_{x\to 0}\frac{arcsen\left(x\frac{1}{\sqrt{x^2+1}}-\sqrt{1-x^2}\frac{x}{\sqrt{x^2+1}}\right)}{x\frac{1}{\sqrt{x^2+1}}-\sqrt{1-x^2}\frac{x}{\sqrt{x^2+1}}}\;\frac{\frac{x}{\sqrt{x^2+1}}(1-\sqrt{1-x^2})}{x^3}\frac{1+\sqrt{1-x^2}}{1+\sqrt{1-x^2}}=$$

$$\lim_{x\to 0}\frac{arcsen\left(x\frac{1}{\sqrt{x^2+1}}-\sqrt{1-x^2}\frac{x}{\sqrt{x^2+1}}\right)}{x\frac{1}{\sqrt{x^2+1}}-\sqrt{1-x^2}\frac{x}{\sqrt{x^2+1}}}\;\frac{\frac{x}{\sqrt{x^2+1}}(x^2)}{x^3(1+\sqrt{1-x^2})}=$$

$$\lim_{x\to 0}\frac{arcsen\left(x\frac{1}{\sqrt{x^2+1}}-\sqrt{1-x^2}\frac{x}{\sqrt{x^2+1}}\right)}{x\frac{1}{\sqrt{x^2+1}}-\sqrt{1-x^2}\frac{x}{\sqrt{x^2+1}}}\;\frac{\frac{1}{\sqrt{x^2+1}}}{(1+\sqrt{1-x^2})}=(1)\frac{1}{2}=\frac{1}{2}$$

12) $\displaystyle\lim_{x\to+\infty}\frac{2^x-2^{-x}}{2^x+2^{-x}}=\lim_{x\to+\infty}\frac{2^x-\frac{1}{2^x}}{2^x+\frac{1}{2^x}}=\lim_{x\to+\infty}\frac{\frac{2^{2x}-1}{2^x}}{\frac{2^{2x}+1}{2^x}}=\lim_{x\to+\infty}\frac{2^{2x}-1}{2^{2x}+1}=1$

13) $\displaystyle\lim_{x\to-\infty}\frac{2^x-2^{-x}}{2^x+2^{-x}}=\lim_{x\to+\infty}\frac{2^{-x}-2^x}{2^{-x}+2^x}=\lim_{x\to+\infty}-\frac{2^x-2^{-x}}{2^x+2^{-x}}=-1$ Resuelto con anterioridad.

14) $\displaystyle\lim_{x\to 0}(cos\,x+sen\,x)^{\frac{1}{x}}=\lim_{x\to 0}(1-1+cos\,x+sen\,x)^{\frac{1}{x}}=$

$$\lim_{x\to 0}\left[(1+cos\,x+sen\,x-1)^{\frac{1}{cos\,x+sen\,x-1}}\right]^{\frac{cos\,x+sen\,x-1}{x}}=e^{\lim\limits_{x\to 0}\frac{cos\,x-1+sen\,x}{x}}=$$

$$e^{\lim\limits_{x\to 0}\frac{cos\,x-1}{x}+\frac{sen\,x}{x}}=e^{\lim\limits_{x\to 0}\frac{cos\,x-1}{x}\frac{cos\,x+1}{cos\,x+1}+\frac{sen\,x}{x}}=e^{\lim\limits_{x\to 0}\frac{cos^2x-1}{x(cos\,x+1)}+\frac{sen\,x}{x}}=$$

$$e^{\lim\limits_{x\to 0}\frac{-sen^2x}{x(cos\,x+1)}+\frac{sen\,x}{x}}=e^{\lim\limits_{x\to 0}\frac{sen\,x}{x}\frac{-sen\,x}{cos\,x+1}+\frac{sen\,x}{x}}=e^{(1)(0)+1}=e$$

15) $\displaystyle\lim_{x\to\sqrt{2}}\frac{e^{2x^2+x-1}-e^{x^2+x+1}}{x^2-2}=\lim_{x\to\sqrt{2}}\frac{e^x(e^{2x^2-1}-e^{x^2+1})}{x^2-2}$ Sea $u=x^2-2\Rightarrow x^2=u+2$

Si $x\to\sqrt{2}\Rightarrow u\to 0$ Reemplazando queda:

$$\lim_{u\to 0}\frac{e^{\sqrt{u+2}}(e^{2u+3}-e^{u+3})}{u}=\lim_{u\to 0}\frac{e^{\sqrt{u+2}}e^3(e^{2u}-e^u)}{u}=\lim_{u\to 0}\frac{e^{\sqrt{u+2}}e^3(e^{2u}-1-(e^u-1))}{u}=$$

$$\lim_{u\to 0}e^{\sqrt{u+2}}e^3\left(\frac{e^{2u}-1}{u}-\frac{e^u-1}{u}\right)=\lim_{u\to 0}e^{\sqrt{u+2}+3}\left(2\frac{e^{2u}-1}{2u}-\frac{e^u-1}{u}\right)=$$

$$e^{\sqrt{2}+3}(2(1)-(1))=e^{\sqrt{2}+3}$$

16) $\lim\limits_{x\to 0}\dfrac{ln\,(1-x)+sen\,(3x)}{x}=\lim\limits_{x\to 0}\dfrac{ln\,(1-x)}{x}+\dfrac{sen\,(3x)}{x}=\lim\limits_{x\to 0}\dfrac{ln\,(1-x)}{x}+\lim\limits_{x\to 0}3\dfrac{sen\,(3x)}{3x}=$

$3+\lim\limits_{x\to 0}\dfrac{ln\,(1-x)}{x}$ Sea $u=ln\,(1-x)\Rightarrow e^u=1-x\Rightarrow x=1-e^u$

$$\lim_{x\to 0}\frac{ln\,(1-x)}{x}=\lim_{u\to 0}\frac{u}{1-e^u}=\lim_{u\to 0}-\frac{u}{e^u-1}=\lim_{u\to 0}-\left(\frac{e^u-1}{u}\right)^{-1}=-1\Rightarrow$$

$$\lim_{x\to 0}\frac{ln\,(1-x)+sen\,(3x)}{x}=3+(-1)=2$$

17) $\lim\limits_{x\to 0}\dfrac{b^x-a^x}{x}=\lim\limits_{x\to 0}\dfrac{b^x-1-(a^x-1)}{x}=\lim\limits_{x\to 0}\dfrac{b^x-1}{x}-\dfrac{a^x-1}{x}=ln\,b-ln\,a=ln\left(\dfrac{b}{a}\right)$

18) $\lim\limits_{x\to 0}\dfrac{8^x-e^x}{x}$ Usando el ejercicio anterior $\lim\limits_{x\to 0}\dfrac{8^x-e^x}{x}=ln\,8-ln\,e=ln\,8-1$

4. Dada la función $f(x)$ calcular $f(x+h)$ y calcular el límite $\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

1) $f(x)=\sqrt{x}\quad \lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}=\lim\limits_{h\to 0}\dfrac{\sqrt{x+h}-\sqrt{x}}{h}$

$$\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}\;\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}=\lim_{h\to 0}\frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})}=\lim_{h\to 0}\frac{h}{h(\sqrt{x+h}+\sqrt{x})}=$$

$$\lim_{h\to 0}\frac{1}{\sqrt{x+h}+\sqrt{x}}=\frac{1}{\sqrt{x+0}+\sqrt{x}}=\frac{1}{2\sqrt{x}}$$

2) $f(x)=\sqrt[3]{x}\quad \lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}=\lim\limits_{h\to 0}\dfrac{\sqrt[3]{x+h}-\sqrt[3]{x}}{h}$

$$\lim_{h\to 0}\frac{\sqrt[3]{x+h}-\sqrt[3]{x}}{h}\;\frac{\sqrt[3]{(x+h)^2}+\sqrt[3]{x(x+h)}+\sqrt[3]{x^2}}{\sqrt[3]{(x+h)^2}+\sqrt[3]{x(x+h)}+\sqrt[3]{x^2}}=$$

$$\lim_{h\to 0}\frac{x+h-x}{h(\sqrt[3]{(x+h)^2}+\sqrt[3]{x(x+h)}+\sqrt[3]{x^2})}=\lim_{h\to 0}\frac{h}{h(\sqrt[3]{(x+h)^2}+\sqrt[3]{x(x+h)}+\sqrt[3]{x^2})}=$$

$$\lim_{h\to 0}\frac{1}{(\sqrt[3]{(x+h)^2}+\sqrt[3]{x(x+h)}+\sqrt[3]{x^2})}=\frac{1}{(\sqrt[3]{(x+0)^2}+\sqrt[3]{x(x+0)}+\sqrt[3]{x^2})}=\frac{1}{\sqrt[3]{x^2}}$$

3) $f(x)=\dfrac{1}{x}\qquad \lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}=\lim\limits_{h\to 0}\dfrac{\frac{1}{x+h}-\frac{1}{x}}{h}$

$$\lim_{h\to 0}\frac{\frac{x-(x+h)}{x(x+h)}}{h}=\lim_{h\to 0}\frac{\frac{-h}{x(x+h)}}{h}=\lim_{h\to 0}\frac{-1}{x(x+h)}=\frac{-1}{x(x+0)}=\frac{-1}{x^2}$$

4) $f(x)=x^2+2x+6 \quad \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{(x+h)^2+2(x+h)+6-(x^2+2x+6)}{h}$

$$\lim_{h\to 0}\frac{x^2+2xh+h^2+2x+2h+6-x^2-2x-6}{h}=\lim_{h\to 0}\frac{h^2+2xh+2h}{h}=$$

$$\lim_{h\to 0}\frac{h(h+2x+2)}{h}=\lim_{h\to 0}h+2x+2=2x+2$$

5) $f(x)=\frac{ax-b}{bx-a} \quad \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\frac{a(x+h)-b}{b(x+h)-a}-\frac{ax-b}{bx-a}}{h}$

$$\lim_{h\to 0}\frac{\frac{(bx-a)(a(x+h)-b)-(b(x+h)-a)(ax-b)}{(b(x+h)-a)(bx-a)}}{h}=\lim_{h\to 0}\frac{(bx-a)(ax+ah-b)-(bx+bh-a)(ax-b)}{h(bx+bh-a)(bx-a)}=$$

$$\lim_{h\to 0}\frac{(abx^2+abhx-b^2x-a^2x-a^2h+ab)-(abx^2+abhx-a^x-b^2x-b^2h+ab)}{h(bx+bh-a)(bx-a)}=$$

$$\lim_{h\to 0}\frac{(-a^2h)-(-b^2h)}{h(bx+bh-a)(bx-a)}=\lim_{h\to 0}\frac{-a^2-(-b^2)}{(bx+bh-a)(bx-a)}=\frac{-a^2+b^2}{(bx-a)(bx-a)}=$$
$$-\frac{a^2-b^2}{(bx-a)^2}$$

6) $f(x)=\frac{1}{\sqrt{x}} \quad \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}}{h}$

$$=\lim_{h\to 0}\frac{\frac{\sqrt{x}-\sqrt{x+h}}{\sqrt{x(x+h)}}}{h}=\lim_{h\to 0}\frac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x(x+h)}}\;\frac{\sqrt{x}+\sqrt{x+h}}{\sqrt{x}+\sqrt{x+h}}=\lim_{h\to 0}\frac{x-(x+h)}{h\sqrt{x(x+h)}(\sqrt{x}+\sqrt{x+h})}=$$

$$\lim_{h\to 0}\frac{-h}{h\sqrt{x(x+h)}(\sqrt{x}+\sqrt{x+h})}=\lim_{h\to 0}\frac{-1}{\sqrt{x(x+h)}(\sqrt{x}+\sqrt{x+h})}=$$

$$\frac{-1}{\sqrt{x(x+0)}(\sqrt{x}+\sqrt{x+0})}=\frac{-1)}{\sqrt{x^2}(\sqrt{x}+\sqrt{x})}=-\frac{1}{2x\sqrt{x}}$$

7) $f(x)=\sqrt{x+3} \quad \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\sqrt{x+h+3}-\sqrt{x+3}}{h}$

$$\lim_{h\to 0}\frac{\sqrt{x+h+3}-\sqrt{x+3}}{h}\;\frac{\sqrt{x+h+3}+\sqrt{x+3}}{\sqrt{x+h+3}+\sqrt{x+3}}=\lim_{h\to 0}\frac{x+h+3-(x+3)}{h(\sqrt{x+h+3}+\sqrt{x+3})}=$$

$$\lim_{h\to 0}\frac{h}{h(\sqrt{x+h+3}+\sqrt{x+3})}=\lim_{h\to 0}\frac{1}{\sqrt{x+h+3}+\sqrt{x+3}}=\frac{1}{2\sqrt{x+3}}$$

8) $f(x)=\sqrt{2x^2-x}\quad \lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}=\lim\limits_{h\to 0}\dfrac{\sqrt{2(x+h)^2-(x+h)}-\sqrt{2x^2-x}}{h}$

$$\lim_{h\to 0}\frac{\sqrt{2(x+h)^2-(x+h)}-\sqrt{2x^2-x}}{h}\;\frac{\sqrt{2(x+h)^2-(x+h)}+\sqrt{2x^2-x}}{\sqrt{2(x+h)^2-(x+h)}+\sqrt{2x^2-x}}=$$

$$\lim_{h\to 0}\frac{2(x+h)^2-(x+h)-(2x^2-x)}{h(\sqrt{2(x+h)^2-(x+h)}+\sqrt{2x^2-x})}=\lim_{h\to 0}\frac{2(x^2+2xh+h^2)-x-h-2x^2+x}{h(\sqrt{2(x+h)^2-(x+h)}+\sqrt{2x^2-x})}=$$

$$\lim_{h\to 0}\frac{4xh+2h^2-h}{h(\sqrt{2(x+h)^2-(x+h)}+\sqrt{2x^2-x})}=\lim_{h\to 0}\frac{h(4x+2h-1)}{h(\sqrt{2(x+h)^2-(x+h)}+\sqrt{2x^2-x})}=$$

$$\lim_{h\to 0}\frac{4x+2h-1}{\sqrt{2(x+h)^2-(x+h)}+\sqrt{2x^2-x}}=\frac{4x+2(0)-1}{\sqrt{2(x+(0))^2-(x+(0))}+\sqrt{2x^2-x}}=$$

$$\frac{4x-1}{\sqrt{2x^2-x}+\sqrt{2x^2-x}}=\frac{4x-1}{2\sqrt{2x^2-x}}$$

9) $f(x)=\dfrac{1}{\sqrt{2x-1}}\quad \lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}=\lim\limits_{h\to 0}\dfrac{\frac{1}{\sqrt{2(x+h)-1}}-\frac{1}{\sqrt{2x-1}}}{h}$

$$\lim_{h\to 0}\frac{\frac{\sqrt{2x-1}-\sqrt{2(x+h)-1}}{\sqrt{2(x+h)-1}\,\sqrt{2x-1}}}{h}=\lim_{h\to 0}\frac{\sqrt{2x-1}-\sqrt{2(x+h)-1}}{h\,\sqrt{2(x+h)-1}\,\sqrt{2x-1}}\;\frac{\sqrt{2x-1}+\sqrt{2(x+h)-1}}{\sqrt{2x-1}+\sqrt{2(x+h)-1}}=$$

$$\lim_{h\to 0}\frac{2x-1-(2(x+h)-1)}{h\,\sqrt{2(x+h)-1}\,\sqrt{2x-1}(\sqrt{2x-1}+\sqrt{2(x+h)-1})}=$$

$$\lim_{h\to 0}\frac{2x-1-2x-2h+1}{h\,\sqrt{2(x+h)-1}\,\sqrt{2x-1}(\sqrt{2x-1}+\sqrt{2(x+h)-1}}=$$

$$\lim_{h\to 0}\frac{-2h}{h\,\sqrt{2(x+h)-1}\,\sqrt{2x-1}(\sqrt{2x-1}+\sqrt{2(x+h)-1}}=$$

$$\lim_{h\to 0}\frac{-2}{\sqrt{2(x+h)-1}\,\sqrt{2x-1}(\sqrt{2x-1}+\sqrt{2(x+h)-1})}=$$

$$\frac{-2}{\sqrt{2(x+0)-1}\,\sqrt{2x-1}(\sqrt{2x-1}+\sqrt{2(x+0)-1})}=$$

$$\frac{-2}{\sqrt{2x-1}\,\sqrt{2x-1}(\sqrt{2x-1}+\sqrt{2x-1})}=\frac{-2}{(2x-1)2\sqrt{2x-1}}=\frac{-1}{(2x-1)\sqrt{2x-1}}$$

10) $f(x)=sen\,x\quad \lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}=\lim\limits_{h\to 0}\dfrac{sen\,(x+h)-sen\,x}{h}$

$$\lim_{h\to 0}\frac{sen\,x\cos h+sen\,h\cos x-sen\,x}{h}=\lim_{h\to 0}\frac{sen\,x(\cos h-1)+sen\,h\cos x}{h}=$$

$$\lim_{h\to 0}\frac{sen\,x(\,cos\,h-1)}{h}+\frac{sen\,h\,cos\,x}{h}=\lim_{h\to 0}\frac{sen\,x(\,cos\,h-1)}{h}+\lim_{h\to 0}\frac{sen\,h\,cos\,x}{h}=$$

$$sen\,x\lim_{h\to 0}\frac{cos\,h-1}{h}+cos\,x\lim_{h\to 0}\frac{sen\,h}{h}=sen\,x(0)+cos\,x(1)=cos\,x$$

11) $f(x)=log\,x \quad \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{log\,(x+h)-log\,x}{h}$

$$\lim_{h\to 0}\frac{log\,(\frac{x+h}{x})}{h}=\lim_{h\to 0}\frac{1}{h}\;log\,\left(\frac{x+h}{x}\right)=\lim_{h\to 0}log\,\left(1+\frac{h}{x}\right)^{\dfrac{1}{h}}=\lim_{h\to 0}log\,\left(1+\frac{1}{\frac{x}{h}}\right)^{\dfrac{1}{h}}=$$

$$\lim_{h\to 0}log\,\left(1+\frac{1}{\frac{x}{h}}\right)^{\dfrac{1}{h}\dfrac{x}{x}}=\lim_{h\to 0}log\,\left(\left(1+\frac{1}{\frac{x}{h}}\right)^{\dfrac{x}{h}}\right)^{\dfrac{1}{x}}=log\,\lim_{h\to 0}\left(\left(1+\frac{1}{\frac{x}{h}}\right)^{\dfrac{x}{h}}\right)^{\dfrac{1}{x}}=$$

$$log\,e^{\frac{1}{x}}=\frac{1}{x}\;\;log\,e$$

12) $f(x)=a^x \quad \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{a^{x+h}-a^x}{h}$

$$\lim_{h\to 0}\frac{a^xa^h-a^x}{h}=\lim_{h\to 0}\frac{a^x(a^h-1)}{h}=a^x\lim_{h\to 0}\frac{a^h-1}{h}=a^x\;ln\,a$$

13) $f(x)=tg\,x \quad \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{tg\,(x+h)-tg\,x}{h}$

Como $tg(\alpha-\beta)=\dfrac{tg\alpha-tg\beta}{1+tg\alpha\,tg\beta}\Rightarrow tg\alpha-tg\beta=(1+tg\alpha\,tg\beta)(tg(\alpha-\beta))$

$$\lim_{h\to 0}\frac{tg\,(x+h)-tg\,x}{h}=\lim_{h\to 0}\frac{(1+tg(x+h)\,tg\,x)tg(x+h-x)}{h}=$$

$$\lim_{h\to 0}\frac{(1+tg(x+h)\,tg\,x)\;tg\,h}{h}=\lim_{h\to 0}(1+tg\,(x+h)tg\,x)\;\frac{tg\,h}{h}=(1+tg^2x)(1)=sec^2x$$

14) $f(x)=ln\,x \quad \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{ln\,(x+h)-ln\,x}{h}$

Por ejercicio resuelto con anterioridad $\lim_{h\to 0}\dfrac{ln\,(x+h)-ln\,x}{h}=\dfrac{1}{x}\;ln\,e=\dfrac{1}{x}(1)=\dfrac{1}{x}$

15) $f(x)=ln\,(2x-1) \quad \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{ln\,(2(x+h)-1)-ln\,(2x-1)}{h}$

$$\lim_{h\to 0}\frac{ln\left(\frac{2x+2h-1}{2x-1}\right)}{h}=\lim_{h\to 0}\frac{1}{h}\,ln\left(\frac{2x-1+2h}{2x-1}\right)=\lim_{h\to 0}\frac{1}{h}\,ln\left(1+\frac{2h}{2x-1}\right)=$$

$$\lim_{h\to 0}\,ln\left(1+\frac{2h}{2x-1}\right)^{\frac{1}{h}}=\lim_{h\to 0}\,ln\left(1+\frac{1}{\frac{2x-1}{2h}}\right)^{\frac{2x-1}{2h}\,\frac{2}{2x-1}}=ln\,e^{\lim_{h\to 0}\frac{2}{2x-1}}=$$

$$ln\,e^{\frac{2}{2x-1}}=\frac{2}{2x-1}\,ln\,e=\frac{2}{2x-1}$$

16) $f(x)=sen\,(3x+c)\quad \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{sen\,(3(x+h)+c)-sen\,(3x+c)}{h}$

$$\lim_{h\to 0}\frac{sen\,(3(x+h)+c)-sen\,(3x+c)}{h}=$$

$$\lim_{h\to 0}\frac{2\,sen\,\left(\frac{3(x+h)+c-(3x+c)}{2}\right)\,cos\,\left(\frac{3(x+h)+c+(3x+c)}{2}\right)}{h}=$$

$$\lim_{h\to 0}\frac{2\,sen\,\left(\frac{3x+3h+c-3x-c)}{2}\right)\,cos\,\left(\frac{3x+3h+c+3x+c}{2}\right)}{h}=$$

$$\lim_{h\to 0}\frac{2\,sen\,\left(\frac{3h}{2}\right)\,cos\,\left(\frac{6x+3h+2c}{2}\right)}{h}=\lim_{h\to 0}\frac{sen\,\left(\frac{3h}{2}\right)\,cos\,\left(\frac{6x+3h+2c}{2}\right)}{\frac{h}{2}}=$$

$$\lim_{h\to 0}\frac{3sen\,\left(\frac{3h}{2}\right)\,cos\,\left(\frac{6x+3h+2c}{2}\right)}{\frac{3h}{2}}=\lim_{h\to 0}3\frac{sen\,\left(\frac{3h}{2}\right)}{\frac{3h}{2}}\,cos\,\left(\frac{6x+3h+2c}{2}\right)=$$

$$(3)\,(1)\,cos\,\left(\frac{6x+3(0)+2c}{2}\right)=(3)\,cos\,\left(\frac{6x+2c}{2}\right)=3\,cos\,(3x+c)$$

17) $f(x)=arctg\,x\quad \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{arctg\,(x+h)-arctg\,x}{h}$

Sea $\alpha=arctg\,(x+h)$ y $\beta=arctg\,x\Rightarrow tg\,\alpha=x+h$ y $tg\,\beta=x$

Sea $\alpha-\beta=t\Rightarrow tg\ t=tg\ (\alpha-\beta)=\frac{tg\alpha-tg\beta}{1+tg\alpha\,tg\beta}=\frac{x+h-x}{1+(x+h)x}=\frac{h}{1+x^2+hx}\Rightarrow$

$$tg\ t=\frac{h}{1+x^2+hx}\Rightarrow t=arctg\,\left(\frac{h}{1+x^2+hx}\right)$$

$$\lim_{h\to 0}\frac{arctg\,(x+h)-arctg\,x}{h}=\lim_{h\to 0}\frac{arctg\,\left(\frac{h}{1+x^2+hx}\right)}{h}=$$

$$\lim_{h\to 0}\frac{arctg\,\left(\frac{h}{1+x^2+hx}\right)}{\frac{h}{1+x^2+hx}}\,\frac{1}{1+x^2+hx}=(1)\,\lim_{h\to 0}\frac{1}{1+x^2+hx}=\frac{1}{1+x^2+0x}=\frac{1}{1+x^2}$$

18) $f(x)=\dfrac{1}{\sqrt[3]{x}} \quad \lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}=\lim\limits_{h\to 0}\dfrac{\frac{1}{\sqrt[3]{x+h}}-\frac{1}{\sqrt[3]{x}}}{h}$

$$\lim_{h\to 0}\frac{\sqrt[3]{x}-\sqrt[3]{x+h}}{h\ \sqrt[3]{x+h}\sqrt[3]{x}}=\lim_{h\to 0}\frac{\sqrt[3]{x}-\sqrt[3]{x+h}}{h\ \sqrt[3]{x^2+hx}}=$$

$$\lim_{h\to 0}\frac{\sqrt[3]{x}-\sqrt[3]{x+h}}{h\ \sqrt[3]{x^2+hx}}\ \frac{\sqrt[3]{x^2}+\sqrt[3]{x(x+h)}+\sqrt[3]{(x+h)^2}}{\sqrt[3]{x^2}+\sqrt[3]{x(x+h)}+\sqrt[3]{(x+h)^2}}=$$

$$\lim_{h\to 0}\frac{x-x-h}{(h\ \sqrt[3]{x^2+hx})(\sqrt[3]{x^2}+\sqrt[3]{x(x+h)})+\sqrt[3]{(x+h)^2}}=$$

$$\lim_{h\to 0}\frac{-h}{(h\ \sqrt[3]{x^2+hx})(\sqrt[3]{x^2}+\sqrt[3]{x(x+h)})+\sqrt[3]{(x+h)^2}}=$$

$$\lim_{h\to 0}\frac{-1}{(\sqrt[3]{x^2+hx})(\sqrt[3]{x^2}+\sqrt[3]{x(x+h)})+\sqrt[3]{(x+h)^2}}=\frac{-1}{(\sqrt[3]{x^2})(3\sqrt[3]{x^2})}=\frac{-1}{3\sqrt[3]{x^4}}$$